The Effects of Carry Distance, Takeoff Angle, Friction Value, and Horizontal Speed Loss Upon First Ground Contact on Pedestrian / Cyclist Crash Reconstruction.

Mike W. Reade, CD, Andrew S. Rich, BSME, ACTAR

Abstract

A traffic crash involving a pedestrian / cyclist presents a challenging investigation for any traffic crash reconstructionist. Well-established methods [1, 2] have provided investigators with mathematical solutions to estimate the speed required to project a pedestrian / cyclist from the area of impact to a final rest position. On-going research [3] continues to validate these methods as reliable.

A crash test dummy is struck by vehicles travelling at various speeds. Electronic equipment such as a Vericom Performance Computer and/or a Doppler radar unit, is used to accurately monitor the vehicle's speed.

The crash test dummy's post-impact distance (known as the throw distance) is measured after each crash test experiment. Class participants are positioned to accurately mark the first contact point on the ground at the end of the airborne phase. The coefficient of friction between the crash test dummy and the test surface is determined by dragging the crash test dummy along the test surface where tests are performed.

The crash test experiments are videotaped at 90° to the motion of the vehicle. Post-impact analysis is useful to measure the crash test dummy takeoff angles and the carry distance as the airborne phase commences.

This research [3] looked at 139 crash test experiments which include both wrap and forward projection trajectories [4]. The crash test dummy is struck standing alone and standing while holding a bicycle or seated on a bicycle.

The analysis of these experiments helps further discuss such topics as; carry distance, pedestrian friction value, pedestrian takeoff angles, and horizontal speed loss upon ground impact.

Introduction

This paper presents a summary of crash tests experiments [3] staged during pedestrian/bicycle crash investigation courses and conferences. The result of these experiments can assist investigators to deal with issues such as; carry distance, friction value, takeoff angles, and horizontal speed loss upon ground impact.

Additional research [5] suggests the pedestrian is carried some distance before the airborne phase commences. That research also shows the pedestrian undergoes a sudden loss of horizontal speed when first contact is made with the ground as the sliding or tumbling phase occurs. This paper confirms that research [3, 5] to be accurate and provides a summary of the data to support these findings.

Current Issues

Many formulas rely upon the pedestrian's (cyclist's) post-impact throw distance while others require additional data such as; the pedestrian's (cyclist's) sliding friction value, the vertical height of the pedestrian's (cyclist's) center-of-mass, and/or other measurements. Regardless what formula is used, the topics often discussed are what values to use for the carry distance, takeoff angles, and friction values.

The formulae [1,2] used to analyze the crash test experiments in this research [3] have been widely used and accepted in the crash reconstruction community. The Searle formulae provide a reliable method to establish a speed necessary to project a pedestrian (cyclist) from the impact area to its final rest. They are:

Searle Minimum Formula:

$$V_{min} = \sqrt{\frac{2\mu g S}{1+\mu^2}} \tag{1}$$

Searle Maximum Formula:

$$V_{max} = \sqrt{2\mu g S} \tag{2}$$

Searle Angle Formula:

$$V = \frac{\sqrt{2\mu g S}}{\cos\theta + (\mu \times \sin\theta)} \tag{3}$$

where μ is the actual friction value which exists between the pedestrian (cyclist) and the surface on which it is sliding or tumbling, g is the gravitational acceleration ($32.2\ fps^2\ or\ 9.81\ m/s^2$), S is the total throw distance the pedestrian (cyclist) travels from impact to final rest, θ is the pedestrian (cyclist) takeoff angle in degrees, and V_{min}, V_{max}, or V represents the projectile's speed, expressed as feet-per-second (fps) or as meters-per-second (m/s), required to project the pedestrian (cyclist) from impact to final rest.

Generally speaking, investigators are able to determine the area of impact and the pedestrian's (cyclist's) final rest location. Not quite so easy to determine is the pedestrian's (cyclist's) carry distance or takeoff angle.

Although some formulae require the investigator to use a friction value, some formulae make an assumption for the friction value of a sliding or tumbling pedestrian (cyclist). It is best for investigators to determine the pedestrian's (cyclist's) friction value by conducting at-scene testing or by making reference to previous research in this area [1, 2, 3, 5, 6]. This paper discusses these topics and how each may affect the overall final calculations. The mean values and the sample standard deviation (s.d.) values are reported for each topic.

Pedestrian (Cyclist) Experiments

As a part of the Pedestrian and Bicycle Crash Investigation and Advanced Pedestrian/Bicycle Crash Investigation courses, the Institute of Police Technology and Management (IPTM) conducts a series of pedestrian (cyclist) crash experiments.

Participants are engaged with preparing for each experiment as well as collecting and analyzing the physical evidence or data resulting from each test.

The pedestrian's (cyclist's) total throw distance, vertical height to center-of-mass, and first ground contact location is measured for each experiment. Slow-motion video is captured at a positon 90-degrees to the motion of the vehicle.

These measurements and data help analyze each experiment and form the basis of this research.

This allows each participant with an opportunity to apply accepted reconstruction methods to analyze pedestrian (cyclist) crash tests and compare their results to instrumented vehicle speeds.

Experiment Results

Pedestrian (Cyclist) Carry Distance

Previous research by Searle [5] suggests the pedestrian is carried a brief period before the airborne phase commences. The carry distance referred to as a standard value in that research is 0.8 meters (2.62 feet).

Appendix 1 shows a typical pedestrian (cyclist) versus vehicle crash diagram and the derivation of the throw distance formulae [5]. (Reproduced with permission.)

During post-crash analysis, the carry distance is measured using a software package called, cSwing [1]. To measure the pedestrian (cyclist) carry distance in this research [3], slow-motion video is recorded at 90-degrees to the motion of the vehicle.

Once the pedestrian (cyclist) carry distance has been measured, an adjustment to the total throw distance is required. This adjustment [5] is performed by reducing the total throw distance by the carry distance. That is:

Searle Carry Distance Adjustment Formula:

$$V_{min} = \sqrt{\frac{2\mu g(S-d)}{1+\mu^2}} \tag{4}$$

where μ is the actual friction value which exists between the pedestrian (cyclist) and the surface on which it is sliding or tumbling, g is the gravitational acceleration ($32.2\ fps^2\ or\ 9.81\ m/s^2$), S is the total throw distance the pedestrian (cyclist) travels from impact to final rest, d is the pedestrian (cyclist) carry distance in feet or meters, and V_{min} represents the speed, expressed as feet-per-second (fps) or as meters-per-second (m/s), required to project the pedestrian (cyclist) from impact to final rest.

Table 1 shows a summary of the carry distance values for; pedestrian-only, cyclist, wrap and forward projection crash test experiments [3]. The average carry distance for 126 wrap trajectory tests is 3.90 feet / 1.19 meters (s.d. = 1.63 feet / 0.50 meters). The average carry distance for 8 forward projection trajectory tests is 4.70 feet / 1.43 meters (s.d. = 2.75 feet / 0.84 meters).

This research confirms previous research [5] that it would be appropriate to reduce the pedestrian (cyclist) throw distance by a carry distance value. It is equally important to mention that a small reduction to the throw distance for carry distance will slightly

[1] See more software information at: http://www.cswing.com

reduce the speed calculation by ~ 1 mph (~ 1.6 km/h) over a distance of 150 feet (45.72 meters), but it does not significantly reduce the final calculation.

Pedestrian (Cyclist) Takeoff Angle

Deciding what pedestrian (cyclist) takeoff angle to use has been discussed for many years and presents a challenge for reconstructionists to decide what is an appropriate value to use. In fact, is it necessary to quantify a pedestrian (cyclist) takeoff angle since investigators are not able to measure this angle in the first place?

Similar to measuring the pedestrian's (cyclist's) carry distance, the post-crash analysis of slow-motion video is performed using the cSwing software so the pedestrian (cyclist) takeoff angle can be measured.

Table 2 shows a summary of the takeoff angle values for; pedestrian-only, cyclist, and wrap crash test experiments [3]. The average takeoff angle for 94 pedestrian-only tests is 6.35 degrees (s.d. = 3.25 degrees). The average takeoff angle for 32 cyclist tests is 6.25 degrees (s.d. = 3.57 degrees), and the average takeoff angle for 126 wrap tests is 6.33 degrees (s.d. = 3.32 degrees).

Equation 3 requires a pedestrian (cyclist) takeoff angle to calculate a result. However, determining a takeoff angle is similar to determining a pedestrian (cyclist) carry distance, it is a difficult value to establish. This research can assist investigators who must use a pedestrian (cyclist) takeoff angle as part of their reconstruction.

Pedestrian (Cyclist) Friction Values

The pedestrian (cyclist) friction value to use for your reconstruction is the actual friction value present between the pedestrian (cyclist) and the crash scene surface on which it slides or tumbles [1, 3, 6]. To measure this value, an alternative method [5] using a weighted sandbag wrapped in similar pedestrian (cyclist) clothing can provide reliable friction values. This technique is similar to many and known as "drag sled" testing to determine the friction value.

Table 3 shows a summary of the friction values as part of this research. The average friction value for 139 experiments [3] is 0.61 (s.d. = 0.09). These experiments do not include the low-friction, or winter road surfaces. In comparison, these results differ little to previous research values.

In 2011, Reade [9] conducted pedestrian low-friction surface testing on wet asphalt (0.58), on slush/wet asphalt (0.53) and on packed snow/asphalt (0.45) conditions. In 2014, Sullenberger [6] conducted low-friction tests on rough ice (0.237), on smooth ice (0.281), on loose-packed snow (0.405), on 6" deep loose snow (0.549) and on slush-asphalt (0.330) surfaces. The results of these studies further support the need to use a lower friction value on low-friction, slippery, or winter road conditions.

Horizontal Speed Loss Upon First Ground Contact

Until recently investigators have taken the position that the pedestrian (cyclist) does not lose any horizontal speed when the pedestrian (cyclist) first contacts the ground and the calculated speeds for the airborne phase will be equivalent to the sliding/tumbling phase.

Earlier research by Searle [1, 5] observed there was a loss of horizontal speed for the pedestrian (cyclist) during the ground impact and before the start of the sliding/tumbling phase.

To confirm this, research by Becker/Reade [3] shows there is indeed a sudden loss of horizontal speed upon first ground impact. The process of calculating this speed loss is outlined in previous research [5]. That is:

Vertical Velocity on Landing:

$$\bar{v} = \sqrt{v^2 + 2gH} \tag{5}$$

where v^2 is the pedestrian's (cyclist's) vertical velocity on takeoff, or start of airborne phase, g is the gravitational acceleration ($32.2\ fps^2\ or\ 9.81\ m/s^2$), H is the vertical height of the pedestrian's (cyclist's) center-of-mass at impact in feet or meters, and $\bar{v}$ represents the vertical velocity of the pedestrian (cyclist) on landing and expressed as feet-per-second (fps) or as meters-per-second (m/s).

In *Equation 5*, the upward vertical velocity component (v) on takeoff can be rewritten as:

$$v = v_o \times \sin\theta \tag{6}$$

and v^2 in *Equation 5* becomes:

$$v^2 = v_o^2 \times \sin^2\theta \tag{7}$$

Substitute *Equation 7* into *Equation 5*:

$$\bar{v} = \sqrt{v_o^2 \times \sin^2\theta + 2gH} \tag{8}$$

where v_o^2 is the pedestrian's (cyclist's) takeoff velocity, or start of airborne phase, $\sin^2\theta$ is the sine of the pedestrian's (cyclist's) takeoff angle in degrees, g is the gravitational acceleration ($32.2\ fps^2\ or\ 9.81\ m/s^2$), H is the vertical height of the pedestrian's (cyclist's) center-of-mass at impact in feet or meters, and $\bar{v}$ represents the vertical velocity of the pedestrian (cyclist) on landing and expressed as feet-per-second (fps) or as meters-per-second (m/s).

NOTE:

Where the crash investigation involves a forward projection trajectory or the takeoff angle is 0 degrees, v^2 becomes zero (0) and *Equation 5 and 8* become:

$$\bar{v} = \sqrt{2gH} \tag{9}$$

Once the vertical velocity of the pedestrian (cyclist) on landing has been determined, the horizontal speed loss upon first ground contact can be calculated by [5]:

Horizontal Speed Loss on Landing:

$$Speed\ Loss\ on\ Landing = \mu\bar{v} \tag{10}$$

and:

$$Speed\ Loss\ on\ Landing = \mu\sqrt{2gH} \tag{11}$$

where μ is the pedestrian's (cyclist's) sliding or tumbling friction value, g is the gravitational acceleration ($32.2\ fps^2\ or\ 9.81\ m/s^2$), H is the vertical height of the pedestrian's (cyclist's) center-of-mass at impact in feet or meters, and $\bar{v}$ represents the vertical velocity of the pedestrian (cyclist) on landing and expressed as feet-per-second (fps) or as meters-per-second (m/s). The results represent the horizontal speed loss ($fps\ or\ m/s$) when the pedestrian (cyclist) first strikes the ground before the sliding or tumbling phase starts.

If an investigator can determine the pedestrian's (cyclist's) total sliding/tumbling distance, the vertical height of center-of-mass, and the takeoff angle, then the calculated horizontal speed loss can be **added** to the pedestrian's / cyclist's slide to stop speed analysis.

Developed from the Searle research [1, 2], Hague [12] discusses how to best deal with the loss of horizontal speed during the ground impact and obtain better results rather than simply ignore the airborne phase.

When a takeoff angle is unknown, as is the case most of the time, investigators can still calculate a projectile speed which is bracketed by the V_{min} and V_{max} equations [1, 2. 5]. Although the calculation does not account for all the horizontal projectile speed, it will account for a good portion of the projectile's speed.

The Becker/Reade [3] crash test experiments did document the takeoff angles and allowed for the calculation of the horizontal speed loss as the crash test dummy first contacted the surface.

Table 4 shows a summary of the horizontal speed loss upon first ground contact for: wrap and forward projection crash test experiments [3]. The average horizontal speed loss upon first ground contact for 32 cyclist crash tests is 6.54 mph/10.52 km/h (s.d. = 1.11 mph/1.78 km/h). The average horizontal speed loss upon first ground contact for 126 wrap crash tests is 6.53 mph/10.51 km/h (s.d. = 1.06 mph/1.70 km/h). The average horizontal speed loss upon first ground contact for 8 forward projection crash tests is 5.20 mph/8.37 km/h (s.d. = 1.11 mph/1.79 km/h). More testing in forward projection experiments is required.

These experiments [3] confirm there is a sudden loss of horizontal speed for the pedestrian (cyclist) before the onset of sliding or tumbling along the ground.

Pedestrian Drop Testing

Additional experiments [3] were conducted by dropping the crash test dummies from a moving vehicle. Here the takeoff angle is zero (0) degrees and represents a forward projection trajectory. The initial crash test dummy speed becomes equivalent to the speed of the moving vehicle.

The vertical drop height of the moving crash test dummy ranged between 3.41 feet (1.04 meters) and 5.50 feet (1.68 meters). The crash test dummy's slide to stop speed is compared to the vehicle speed when the crash test dummy is released. As expected, the crash test dummy's slide to stop speed is **always less than** the speed of the moving vehicle. These experiments [3] confirm there is a loss of horizontal speed for the pedestrian (cyclist) before the sliding or tumbling phase commences.

Table 4 shows a summary of the drop testing [3] when the crash test dummy first contacts the ground. The average horizontal speed loss upon first ground contact for 39 crash tests is 7.08 mph/11.39 km/h (s.d. = 0.78 mph/1.26 km/h). There appear to be little difference between the mean values of the horizontal speed loss experiments (6.47 mph/10.41 km/h) and the vertical drop test experiments (7.08 mph/11.39 km/h).

These experiments confirm previous research [5] to explain why the airborne speed is not equivalent to the sliding/tumbling speed without further consideration.

Given a reconstruction where an investigator can only document the pedestrian's /cyclist's total sliding distance, the resulting slide to stop speed from the pedestrian / cyclist analysis **will always be conservative and underestimate the projectile's airborne speed**, unless of course the pedestrian's / cyclist's sliding distance is overestimated. Here, investigators are able to provide a more complete analysis by considering this horizontal speed loss adjustment.

Conclusions

This on-going research [3] shows there is a carry distance that occurs before the airborne phase commences. To account for this investigators can reduce the total throw distance by the carry distance. The final speed calculation will be slightly less if an adjustment for carry distance is considered.

Often, investigators cannot determine a pedestrian's / cyclist's takeoff angle during a pedestrian versus a vehicle crash unless there is nearby video which might allow for such an analysis. Research [3] shows the average takeoff is not significant and that using a takeoff angle value will increase the final speed calculation. Not using a takeoff angle will provide a more conservative approach. If the reconstruction involves a forward projection trajectory, takeoff angles are no longer an issue.

For collisions occurring on normal road surfaces, the differences in the pedestrian's (cyclist's) friction value do not significantly affect the final speed calculations, unless of course there is a low-friction surface as shown by the Sullenberger research [6] conducted under winter conditions.

If one is going to make adjustments to the throw distance because of pedestrian carry, the center-of-mass height above the ground at impact [5] or to use a pedestrian friction value of 0.70, the V_{min} and V_{max} equations [1, 2, 5] do a good job to bracket projectile speeds in real world crashes.

Being able to determine the pedestrian's / cyclist's horizontal speed loss upon first ground contact is helpful when the area of impact cannot be established and investigators are only able to determine the pedestrian's / cyclist's sliding/tumbling distance. After calculating the pedestrian's / cyclist's horizontal speed loss upon first ground contact, **ADD** it to the slide to stop speed analysis.

This adjustment **is only required** when the area of impact is unknown. The sliding/tumbling speed on its own is conservative and underestimates the pedestrian (cyclist) projectile speed.

As research in this area continues, new analytical techniques and tools [10] are becoming very helpful to show the dynamics of pedestrian crashes. Scurlock[2] [11] shows the use of the Virtual CRASH [13] simulator to model pedestrian impacts has potential to faithfully reproduce expected projectile behaviors.

About the Authors

Mike W. Reade, CD is the owner of Forensic Reconstruction Specialists Inc., a collision reconstruction consulting firm located in Moncton, New Brunswick Canada. He is an Adjunct Instructor with the Institute of Police Technology and Management – University of North Florida based out of Jacksonville, Florida. His website is www.frsi.ca. He can be reached at reademw@gmail.com.

Andrew S. Rich, BSME, ACTAR is the owner of Rich Consulting LLC, which provides accident reconstruction consulting and training services based in Fairlawn, Ohio USA. He is an Adjunct Instructor with the Institute of Police Technology and Management – University of North Florida based out of Jacksonville, Florida. His website is www.rich-llc.com. He can be reached at andy.s.rich@gmail.com.

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References

[1] Searle, J.A., Searle, A. (1983) *"The Trajectories of Pedestrians, Motorcycles, Motorcyclists, etc., Following a Road Accident."* SAE Technical #831622.

[2] Searle, J.A. (1993) *"The Physics of Throw Distance in Accident Reconstruction."* SAE Technical #9306759.

[3] Becker, T.L., Reade, M.W. (2008 to 2016) *"Analysis of Controlled Pedestrian / Cyclist Crash Testing Data."* IPTM-UNF Pedestrian/Bicycle Crash Investigation Courses.

[4] Ravani, B., Brougham, D., Mason, R.T. (1981) *"Pedestrian Post-Impact Kinematics and Injury Patterns."* SAE Technical #811024.

[5] Searle, J.A. (2009) *"The Application of Throw Distance Formulae."* IPTM Special Problems in Traffic Crash Reconstruction, Orlando, Florida.

[6] Sullenberger, G. A. (2014) *"Pedestrian Impact on Low Friction Surface."* SAE Technical #2014-01-0470.

[7] Craig, A. (1999) *"Bovington Test Results."* Impact Vol 8, No. 3, pp 83 – 85. ITA1, England, 1999.

[2] Research paper available at: http://arxiv.org/abs/1512.00790

[8] Hill, G. S. (1994) *"Calculations of Vehicle Speed from Pedestrian Throw."* Impact Vol. 4, No. 1, pp 18 – 20, ITA1, England, 1994.

[9] Reade, M. W. (2011) *"CATAIR Atlantic Region Pedestrian Crash, Drop & Friction Testing."* Riverview, New Brunswick, Canada, 2011.

[10] PEDBIKE 2000 Plus *"Pedestrian and Bicycle Specialty Software."* Web Site: http://www.frsi.ca/ , Riverview, New Brunswick, Canada, 2000 - Present.

[11] Becker, T., Reade, M., Scurlock, B. (2015) *"Simulations of Pedestrian Impact Collisions with Virtual CRASH 3 and Comparisons with IPTM Staged Tests."* Cornell University Library, arXiv: 1512.00790.

[12] Hague, D.J. (2001) *"Calculation of Impact Speed from Pedestrian Slide Distance."* Metropolitan Laboratory Forensic Science Service, ITAI Conference.

[13] Virtual CRASH *"Software for Accident Reconstruction."* Web Site: http://www.vcrashusa.com/ , North America and Caribbean Distributor, Newberry, Florida, 2016.

Table 1 – Summary of Carry Distance Research. [3]

Experiment Type	Mean	Low	High	Std. Dev. (s)	No. of Tests
All Crash Tests	3.91 ft (1.19 m)	0.0 ft (0.00 m)	9.61 ft (2.93 m)	1.74 ft (0.53 m)	139
Pedestrian Only Tests	3.86 ft (1.17 m)	1.73 ft (0.53 m)	9.61 ft (2.93 m)	1.54 ft (0.47 m)	94
Cyclist Tests	4.03 ft (1.23 m)	1.65 ft (0.50 m)	9.57 ft (2.92 m)	1.91 ft (0.58 m)	32
Wrap Trajectory Tests	3.90 ft (1.19 m)	1.65 ft (0.50 m)	9.61 ft (2.93 m)	1.63 ft (0.50 m)	126
Forward Projection Tests	4.70 ft (1.43 m)	1.00 ft (0.30 m)	9.60 ft (2.93 m)	2.75 ft (0.84 m)	8

Table 2 – Summary of Take-Off Angle Research. [3]

Experiment Type	Mean	Low	High	Std. Dev. (s)	No. of Tests
Pedestrian Only Tests	6.35 degrees	1.00 degree	17 degrees	3.25 degrees	94
Cyclist Tests	6.25 degrees	1.00 degree	16 degrees	3.57 degrees	32
Wrap Trajectory Tests	6.33 degrees	1.00 degree	17 degrees	3.32 degrees	126

Table 3 - Summary of Pedestrian Friction Value Research. [3]

Clothing / Surface Friction Values	Mean	Low	High	Std. Dev. (s)
Hill [8] (Jacket, Trousers, Nylon, Leather, Jeans, Woolen suit)	0.695	0.567	.750	0.073
Bovington [7] (Nylon, Leather, Fabric, Jeans)	0.584	0.532	0.633	0.039
Searle [5] (Sandbag on Various Surfaces)	0.63	0.30	0.78	0.134
Becker/Reade [3] (Cotton, Nylon, Woolen, Jeans)	0.590	0.440	0.690	0.085
Sullenberger [6] (Mean Low-Friction, Winter Conditions)	0.36	0.237	0.549	0.122
Reade [9] (Low-Friction, Winter Conditions)	0.52	0.45	0.58	0.03
Research Average Values (* Summary of above.)	* 0.56 (avg.)	* 0.42 (avg.)	* 0.66 (avg.)	0.06

Table 4 – Summary of Horizontal Speed Loss Upon First Ground Contact & Vertical Drop Testing Research. [3]

Experiment Type	Mean	Low	High	Std. Dev. (s)	No. of Tests
All Crash Tests	6.47 mph (10.41 km/h)	3.18 mph (5.13 km/h)	10.85 mph (17.46 km/h)	1.11 mph (1.78 km/h))	139
Pedestrian Only Tests	6.53 mph (10.51 km/h)	4.38 mph (7.05 km/h)	10.85 mph (17.46 km/h)	1.04 mph (1.68 km/h)	94
Cyclist Tests	6.54 mph (10.52 km/h)	4.51 mph (7.25 km/h)	9.30 mph (14.97 km/h)	1.11 mph (1.78 km/h)	32
Wrap Trajectory Tests	6.53 mph (10.51 km/h)	4.38 mph (7.05 km/h)	10.85 mph (17.46 km/h)	1.06 mph (1.70 km/h)	126
Forward Projection Tests	5.20 mph (8.37 km/h)	3.18 mph (5.13 km/h)	6.51 mph 10.48 km/h)	1.11 mph (1.79 km/h)	8
Crash Test Dummy Drop Testing	7.08 mph (11.39 mph)	5.32 mph (8.56 km/h)	8.21 mph (13.22 km/h)	0.78 mph (1.26 km/h)	39

Chart 1: Searle Minimum Equation Results v. Test Speed - Wrap Experiments. [3]

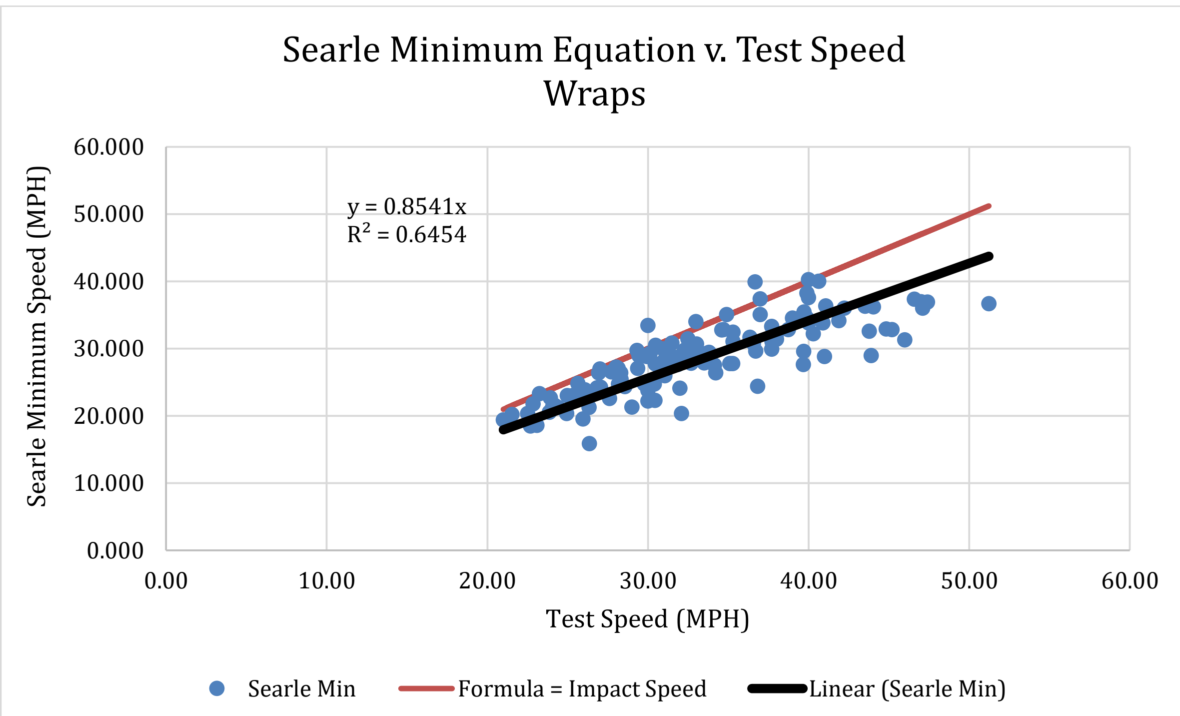


Chart 2: Searle Maximum Equation Results v. Test Speed - Wrap Experiments. [3]

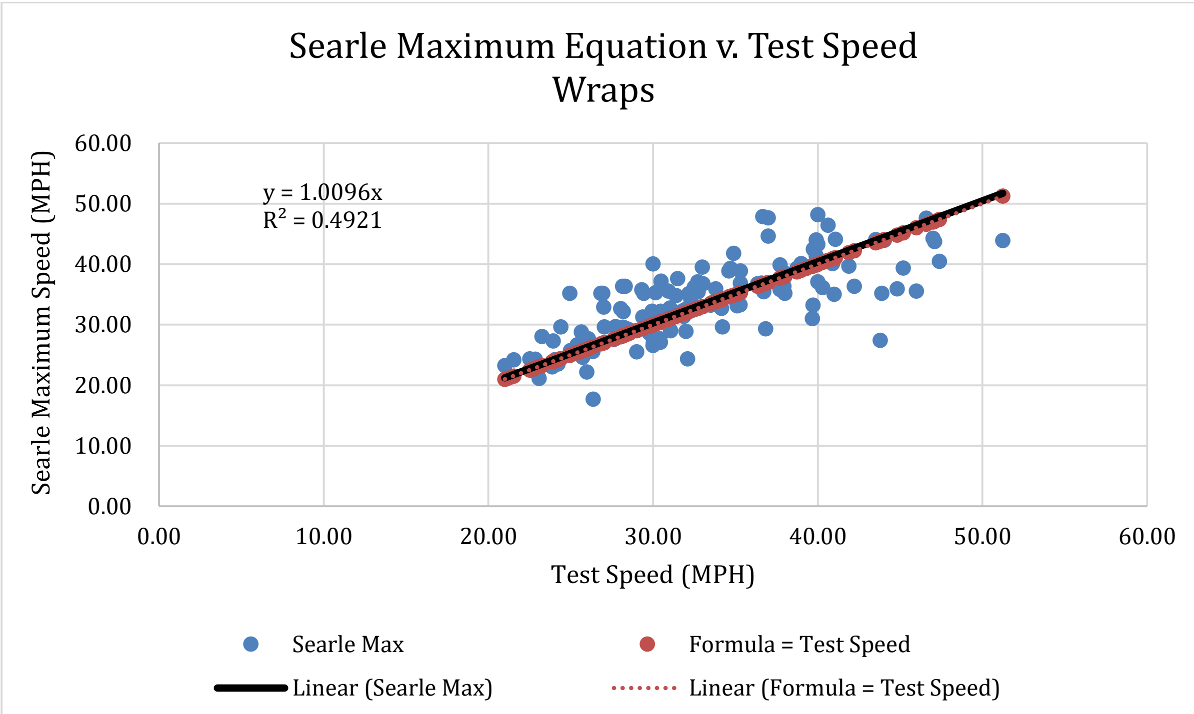


Chart 3: Searle Minimum Equation Results v. Test Speed - Forward Projection Experiments. [3]

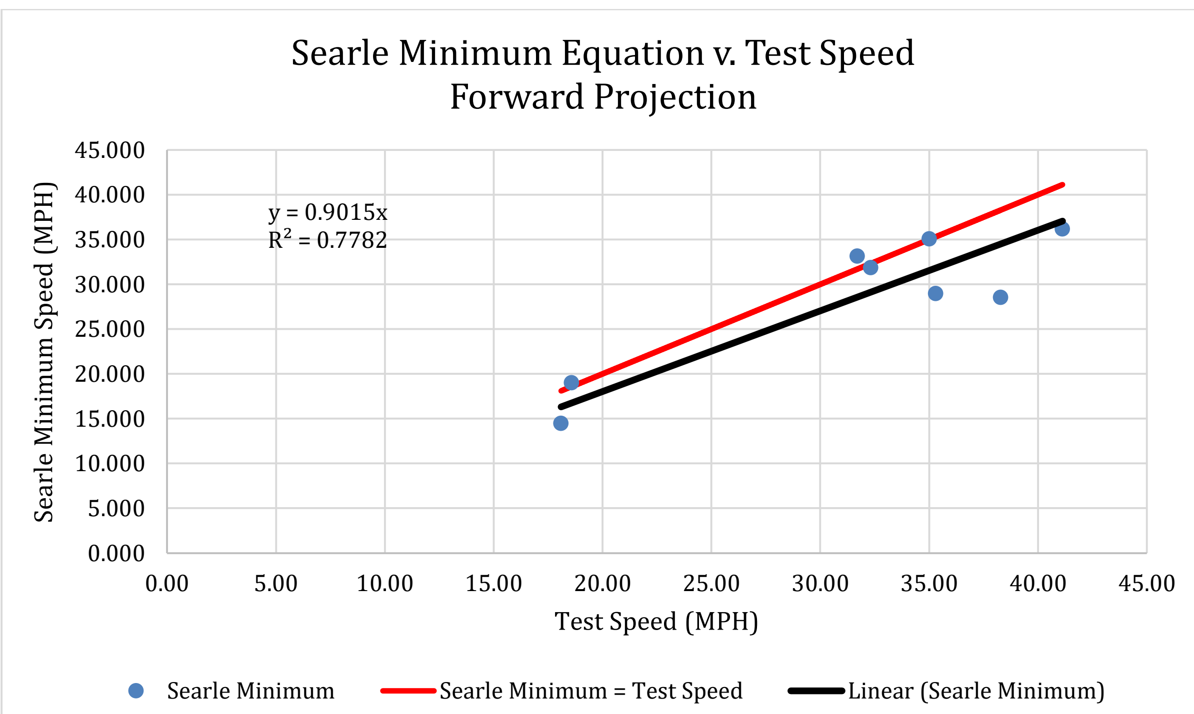
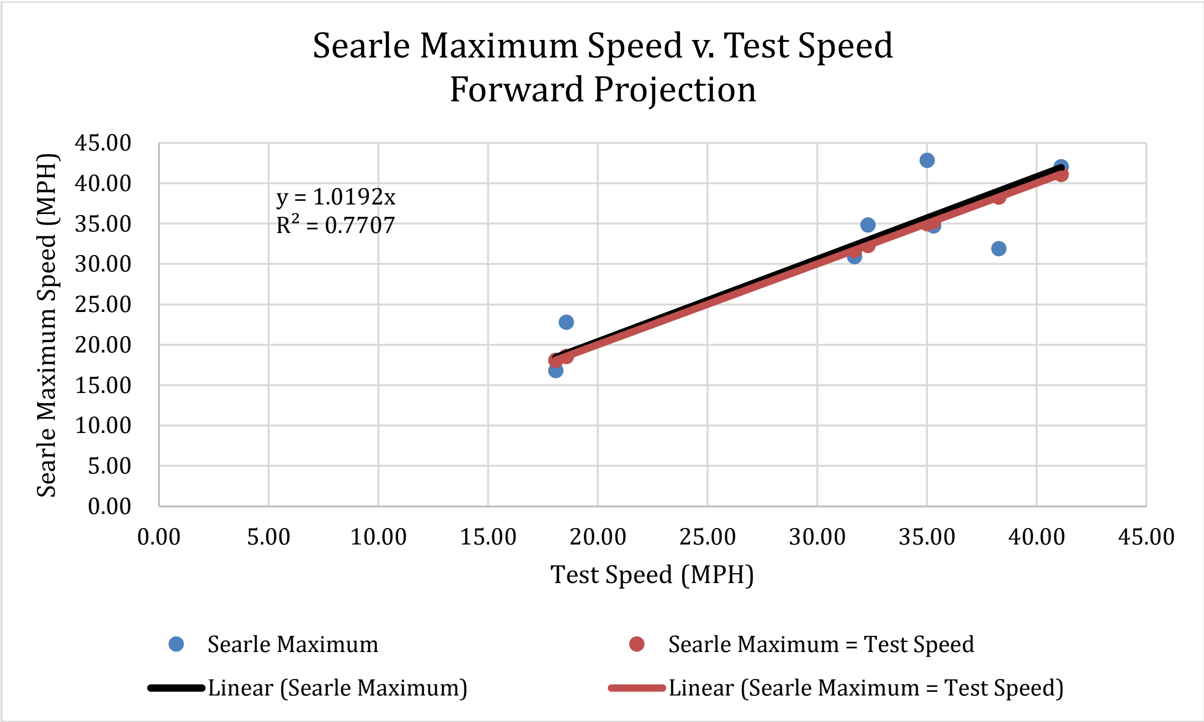


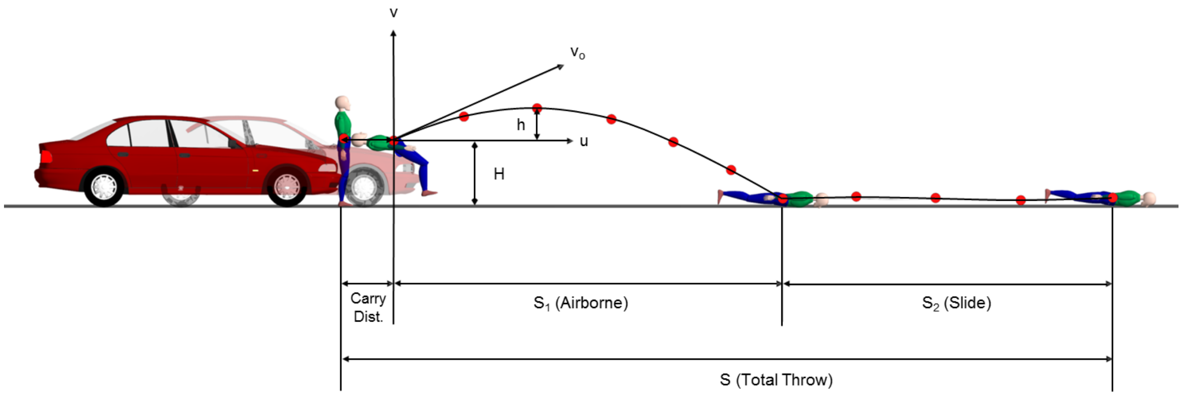
Chart 4: Searle Maximum Equation Results v. Test Speed Forward Projection Experiments. [3]



NOTE:

This research includes a limited number of forward projection crash test experiments. Therefore, more forward projection testing is required.

Appendix 1: Derivation of throw distance formulae [5] – (Reproduced with permission).



Variables:

v	=	Vertical velocity at takeoff in feet-per-second (fps) or meters-per-second (m/s) and $v = V \sin\theta$.
v_o or V	=	Pedestrian's original takeoff velocity in feet-per-second (fps) or meters-per-second (m/s).
u	=	Horizontal velocity at takeoff in feet-per-second (fps) or meters-per-second (m/s) and $u = V \cos\theta$.
h	=	Maximum height above takeoff height measured upward, or positively in feet (ft) or meters (m) above H.
H	=	Vertical height to pedestrian's center-of-mass measured upward, or positively in feet (ft) or meters (m).
g	=	Gravitational acceleration in feet-per-second^2 (fps^2) or meters-per-second^2 (m/s^2).
$\bar{v}$	=	Vertical velocity at landing in feet-per-second (fps) or meters-per-second (m/s).
t	=	Time to landing in seconds.
μ	=	Pedestrian friction value.
θ	=	Pedestrian's takeoff angle measured between u and v_o or V measured in degrees.

NOTE:

The $Carry\ Dist.$, $S_1(Airborne)$, $S_2(Slide)$, and $S\ (Total\ Throw)$ distances are measured in feet (ft) or meters (m).

Maximum Height Above Takeoff Height:

$$h = \frac{v^2}{2g} \tag{1}$$

Vertical Velocity on Landing:

$$\bar{v} = \sqrt{v^2 + 2gH} \tag{2}$$

Time to Landing:

$$t = \frac{v + \bar{v}}{g} \tag{3}$$

Distance Travelled Before Landing:

$$S_1 = ut = u\left(\frac{v + \bar{v}}{g}\right) \tag{4}$$

Horizontal Speed Loss on Landing:

$$Speed\ Loss\ on\ Landing = \mu\bar{v} \tag{5}$$

Wrap Trajectories:

$$Speed\ Loss\ on\ Landing = \mu\sqrt{v^2 + 2gH} \tag{6}$$

or:

$$Speed\ Loss\ on\ Landing = \mu\sqrt{v_o^2 \times \sin^2\theta + 2gH} \tag{7}$$

Forward Projection Trajectories:

$$Speed\ Loss\ on\ Landing = \mu\sqrt{2gH} \tag{8}$$

Distance Travelled After Landing:

$$S_2 = \frac{(u - \mu\bar{v})^2}{2\mu g} \tag{9}$$

Total Throw Distance:

$$S = S_1 + S_2 \tag{10}$$

$$S = u\left(\frac{v + \bar{v}}{g}\right) + \frac{(u - \mu\bar{v})^2}{2\mu g} \tag{11}$$

$$S = \frac{[(2\mu uv + 2\mu u\bar{v}) + (u^2 - 2\mu u\bar{v} + \mu^2\bar{v}^2)]}{2\mu g} \tag{12}$$

$$S = \frac{u^2 + 2\mu uv + \mu^2 v^2 + 2\mu^2 gH}{2\mu g} \tag{13}$$

$$S = \frac{(u + \mu v)^2}{2\mu g} + \mu H \tag{14}$$

The extra distance travelled is due to the initial height of the pedestrian's (cyclist's) center-of-mass and is equal to μH

Recall that $v = V \sin\theta$ and $u = V \cos\theta$ therefore, Equation 14 can be written as [1, 5]:

$$S = \frac{V^2}{2\mu g}(\cos\theta + \mu\sin\theta)^2 + \mu H \tag{15}$$

$$\frac{2\mu g(S - \mu H)}{(\cos\theta + \mu\sin\theta)^2} = V^2 \tag{16}$$

Rearranged to represent the Searle formula [1, 5] which considers an adjustment for the pedestrian's center-of-mass height at impact and requires a projectile takeoff angle:

$$V = \frac{\sqrt{2\mu g(S - \mu H)}}{(\cos\theta + \mu\sin\theta)} \tag{17}$$

Since reconstructionists normally cannot determine a takeoff angle, an investigator can consider an appropriate value for θ, the projectile's takeoff angle, to establish an upper (maximum) and a lower (minimum) limit for the projection velocity.

To find the maximum value for V, where θ = 0, the Searle Maximum [5] formula becomes:

$$V_{max} = \sqrt{2\mu g(S - \mu H)} \tag{18}$$

To find the minimum value for V, note that:

$$1 - \sin^2\theta - \cos^2\theta = 0 \tag{19}$$

Expand $(\cos\theta + \mu\sin\theta)^2$ in *Equation 16* by putting in extra zero (0) terms::

$$(\cos\theta + \mu\sin\theta)^2 = \cos^2\theta + 2\mu\sin\theta\cos\theta + \mu^2\sin^2\theta + (1 - \sin^2\theta - \cos^2\theta) + \mu^2(1 - \sin^2\theta - \cos^2\theta) \tag{20}$$

$$(\cos\theta + \mu\sin\theta)^2 = 1 + \mu^2 - [\sin\theta - \mu\cos\theta]^2 \tag{21}$$

Because $[\sin\theta - \cos\theta]^2$ is squared and cannot go negative, its minimum value is zero (0). Therefore, the maximum value of $(\cos\theta + \mu\sin\theta)^2$ is:

$$(\cos\theta + \mu\sin\theta)^2 = 1 + \mu^2 \tag{22}$$

After substituting $1 + \mu^2$ for $(\cos\theta + \mu\sin\theta)^2$ in Equation 16, we find the well-known Searle Minimum [1, 2] formula which considers an adjustment for the pedestrian's (cyclist's) height of center-of-mass. [5] That is:

$$V_{min} = \sqrt{\frac{2\mu g(S - \mu H)}{1 + \mu^2}} \tag{23}$$

Appendix 2: Overall Summary of Crash Test Experiments. [3]

"Pedestrian Crash Test Data Summary"														
Date of Summary: **February 15, 2016**	**The following summary is based upon crash test data collected during IPTM Pedestrian/Bicycle Courses and on-going controlled test sessions.**													
Total Number of Crash Tests:	**All Crash Tests**		**Ped Only Tests**		**Cyclist Tests**		**Wrap Tests**		**Forward Proj. Tests**		**Corner Tests**		**Drop Tests (Only)**	
	139 Tests		**94 Tests**		**32 Tests**		**126 Tests**		**8 Tests**		**5 Tests**		**39 Tests**	
Vehicle Impact Speed:	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**
(Average)	33.47	53.86	31.90	51.34	36.66	59.01	33.11	53.28	31.30	50.37	46.00	74.03	34.61	55.70
(Minimum)	18.09	29.11	21.00	33.80	22.70	36.53	21.00	33.80	18.09	29.11	39.00	62.76	26.00	41.84
(Maximum)	52.83	85.02	51.23	82.45	47.40	76.28	51.23	82.45	41.12	66.18	52.83	85.02	53.00	85.30
Pedestrian Throw Distance:	**ft**	**m**	**ft**	**m**	**ft**	**m**	**ft**	**m**	**ft**	**m**	**ft**	**m**	**ft**	**m**
(Average)	63.23	18.70	62.09	18.93	71.33	21.74	64.93	19.79	65.20	19.87	30.17	9.19	N/A	N/A
(Minimum)	12.80	3.79	27.30	8.32	21.00	6.40	21.00	6.40	16.00	4.88	12.80	3.90	N/A	N/A
(Maximum)	122.10	36.12	117.50	35.81	122.10	37.22	122.10	37.22	100.00	30.48	50.83	15.49	N/A	N/A
Pedestrian Friction Value:	**μ**		**μ**		**μ**		**μ**		**μ**		**μ**		**μ**	
(Average)	0.61		0.61		0.60		0.61		0.57		0.63		0.60	
(Minimum)	0.44		0.44		0.44		0.44		0.44		0.44		0.45	
(Maximum)	0.79		0.79		0.79		0.79		0.70		0.70		0.64	
Standard Deviation	0.09		0.09		0.09		0.09		0.07		0.07		0.05	
Pedestrian Takeoff Angle:	**Degrees**		**Degrees**		**Degrees**		**Degrees**		**Degrees**		**Degrees**		**Degrees**	
(Average)	6.35		6.35		6.25		6.33		0.00		7.33		0.00	
(Minimum)	0.00		1.00		1.00		1.00		0.00		0.00		0.00	
(Maximum)	17.00		17.00		16.00		17.00		0.00		10.00		0.00	
Standard Deviation	3.58		3.25		3.57		3.32		0.00		4.56		0.0	

Pedestrian Carry Distance:	ft	m	ft	m	ft	m	ft	m	ft	m	ft	m	ft	m
(Average)	3.91	1.19	3.86	1.18	4.03	1.23	3.90	1.19	4.70	1.43	2.46	0.75	N/A	N/A
(Minimum)	0.00	0.00	1.73	0.53	1.65	0.50	1.65	0.50	1.00	0.30	0.00	0.00	N/A	N/A
(Maximum)	9.61	2.93	9.61	2.93	9.57	2.92	9.61	2.93	9.60	2.93	3.60	1.10	N/A	N/A
Standard Deviation	1.74	0.53	1.54	0.47	1.91	0.58	1.63	0.50	2.75	0.84	1.44	0.44	N/A	N/A
Searle Minimum Speed Results:	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**
(Average)	28.19	45.37	28.05	45.15	29.92	48.14	28.53	45.91	28.41	45.72	19.44	31.29	N/A	N/A
(Minimum)	13.27	21.36	18.58	29.90	15.86	25.52	15.86	25.52	14.48	23.30	13.27	21.36	N/A	N/A
(Maximum)	40.21	64.72	40.21	64.72	40.00	64.37	40.21	64.72	36.20	58.25	26.45	42.57	N/A	N/A
Searle Min Speed - Carry Dist. Results:	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**
(Average)	27.27	43.89	27.12	43.64	29.03	46.71	27.60	44.42	27.34	44.00	18.83	30.30	N/A	N/A
(Minimum)	12.59	20.26	17.84	28.71	15.10	24.30	15.10	24.30	12.59	20.26	13.27	21.36	N/A	N/A
(Maximum)	39.07	62.87	39.07	62.87	39.04	62.82	39.07	62.87	34.94	56.24	25.76	41.46	N/A	N/A
Pedestrian Slide to Stop Results:	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**
(Average)	20.45	32.91	19.68	31.67	21.27	34.23	20.08	32.32	28.26	45.48	17.67	28.43	26.92	43.32
(Minimum)	8.04	12.94	8.05	12.95	9.57	15.39	8.05	12.95	10.52	16.93	15.62	25.15	17.33	27.89
(Maximum)	38.16	61.41	38.20	61.48	30.89	49.71	38.20	61.48	36.37	58.54	21.49	34.59	46.76	75.26
Horizontal Speed Loss on Ground Impact:	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**	**mph**	**km/h**
(Average)	6.47	10.41	6.53	10.51	6.54	10.52	6.53	10.51	5.20	8.37	6.89	11.08	7.08	11.39
(Minimum)	3.18	5.13	4.38	7.05	4.51	7.25	4.38	7.05	3.18	5.13	4.90	7.89	5.32	8.56
(Maximum)	10.85	17.46	10.85	17.46	9.30	14.97	10.85	17.46	6.51	10.48	8.11	13.05	8.21	13.22
Standard Deviation	1.11	1.78	1.04	1.68	1.11	1.78	1.06	1.70	1.11	1.79	1.31	2.10	0.78	1.26
Percentage of Vehicle Speed Acquired:	**%**		**%**		**%**		**%**		**%**		**%**		**%**	
(Searle Min Speed ÷ Vehicle Impact Speed)	86%		88%		82%		87%		91%		43%		98%	